

# Example Candidate Responses

Cambridge  
International  
AS & A Level

## Cambridge International AS and A Level Mathematics

9709

Paper 2

Cambridge Advanced



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International Examinations

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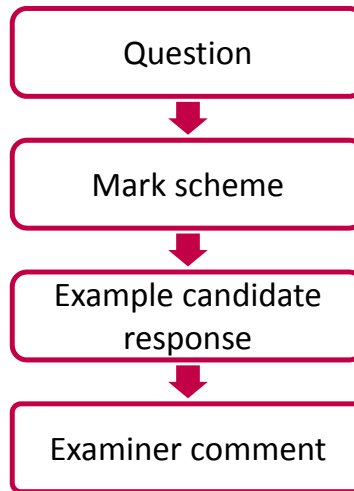
## Introduction

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The main aim of this booklet is to exemplify standards for those teaching Cambridge International AS & A Level Mathematics (9709), and to show how different levels of candidates' performance relate to the subject's curriculum and assessment objectives.

In this booklet candidate responses have been chosen to exemplify a range of answers. Each response is accompanied by a brief commentary explaining the strengths and weaknesses of the answers.

For ease of reference the following format for each component has been adopted:



Each question is followed by an extract of the mark scheme used by examiners. This, in turn, is followed by examples of marked candidate responses, each with an examiner comment on performance. Comments are given to indicate where and why marks were awarded, and how additional marks could have been obtained. In this way, it is possible to understand what candidates have done to gain their marks and what they still have to do to improve them.

Past papers, Examiner Reports and other teacher support materials are available on Teacher Support at <https://teachers.cie.org.uk>

## Assessment at a glance

The 7 units in the scheme cover the following subject areas:

- Pure Mathematics (units P1, P2 and P3);
- Mechanics (units M1 and M2);
- Probability and Statistics (units S1 and S2).

Centres and candidates may:

- take all four Advanced (A) Level components in the same examination session for the full A Level.
- follow a staged assessment route to the A Level by taking two Advanced Subsidiary (AS) papers (P1 & M1 **or** P1 & S1) in an earlier examination session;
- take the Advanced Subsidiary (AS) qualification only.

AS Level candidates take:

Paper 1: Pure Mathematics 1 (P1)
<p><b>1¼ hours</b> About 10 shorter and longer questions 75 marks weighted at 60% of total</p>

plus **one** of the following papers:

Paper 2: Pure Mathematics 2 (P2)	Paper 4: Mechanics 1 (M1)	Paper 6: Probability and Statistics (S1)
<p><b>1¼ hours</b> About 7 shorter and longer questions 50 marks weighted at 40% of total</p>	<p><b>1¼ hours</b> About 7 shorter and longer questions 50 marks weighted at 40% of total</p>	<p><b>1¼ hours</b> About 7 shorter and longer questions 50 marks weighted at 40% of total</p>

Teachers are reminded that the latest syllabus is available on our public website at [www.cie.org.uk](http://www.cie.org.uk) and Teacher Support at <https://teachers.cie.org.uk>

## Paper 2

### Question 1

1 Solve the inequality  $|2x + 1| < |2x - 5|$ . [3]

#### Mark scheme

<b>1 EITHER</b>	State or imply non-modular inequality $(2x+1)^2 < (2x-5)^2$ , or corresponding equation or pair of linear equations Obtain critical value 1  State correct answer $x < 1$	M1 A1 A1
<b>OR</b>	State the critical value $x = 1$ , by solving a linear equation (or inequality) or from a graphical method or by inspection State correct answer $x < 1$	B2 B1 [3]

#### Example candidate response – 1

1)  $|2x+1| < |2x-5|$   
 $(2x+1)^2 < (2x-5)^2$   
 $4x^2 + 4x + 1 < 4x^2 - 10x - 10x + 25$  M1  
 $4x^2 + 4x + 1 < 4x^2 - 20x + 25$  A1  
 $4x + 1 < -20x + 25$  | -24x - 24 > 0  
 ~~$24x > 24$~~  | -24(x-1) > 0  
 ~~$x > 1$~~  |  $x > 1$  A0  
(2)

Total mark awarded = 2 out of 3

## Example candidate response – 2

$$\begin{array}{l}
 1.) \quad |2x+1| < |2x-5| \\
 (2x+1)^2 < (2x-5)^2 \quad \checkmark \\
 4x^2 + 4x + 1 < 4x^2 - 20x + 25 \\
 24x < 24 \\
 x < \frac{24}{24} \quad \text{or} \quad x < 0.96 \quad (\text{3.s.f.})
 \end{array}$$

M1  
A0  
A0  
①

Total mark awarded = 1 out of 3

## Examiner comment

Both candidates chose to solve the inequality using  $(2x+1)^2 < (2x-5)^2$ . This method was that most commonly used by candidates.

Candidate 1 was able to expand out the brackets and simplify the resulting terms correctly, going on to obtain a correct critical value, whereas candidate 2 made an arithmetic slip in the simplification, thus obtaining an incorrect critical value and no further marks.

Candidate 1 was unable to obtain the correct range of values as the negative nature of the right hand side of the inequality was not dealt with correctly.

## Question 2

- 2 The curve with equation  $y = \frac{\sin 2x}{e^{2x}}$  has one stationary point in the interval  $0 \leq x \leq \frac{1}{2}\pi$ . Find the exact  $x$ -coordinate of this point. [4]

## Mark scheme

- 2 Use quotient rule or product rule, correctly M1  
Obtain correct derivative in any form A1  
Equate derivative to zero and solve for  $x$  M1  
Obtain  $x = \frac{\pi}{8}$  A1 [4]



## Example candidate response – 1

$$2) \quad y = \frac{\sin 2x}{e^{2x}}$$

$$u = \sin 2x \quad v = e^{2x}$$

$$\frac{du}{dx} = 2 \cos 2x \quad \frac{dv}{dx} = e^{2x} \cdot 2$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{(v)^2}$$

$$= \frac{e^{2x} \cdot 2 \cos 2x - \sin 2x \cdot 2e^{2x}}{(e^{2x})^2}$$

$$= \frac{2 \cos 2e^{2x} x - 2e^{2x} \sin 2x}{(e^{2x})^2}$$

M1

$$= \frac{e^{2x} \cdot 2 \cos 2x - \sin 2x \cdot 2e^{2x}}{(e^{2x})^2}$$

A1

$$= \frac{2 \cos 2e^{2x} x - 2e^{2x} \sin 2x}{(e^{2x})^2}$$

Example candidate response – 1, continued

$2e^{2x} [2\cos 2x]$	<p>Nothing is to be written in this margin</p>
$= 2e^{2x} [2\cos 2x - \sin 2x]$	
<p>At stationary pt <math>\frac{dy}{dx} = 0</math></p>	
$2e^{2x} [2\cos 2x - \sin 2x] = 0$	
$2e^{2x} = 0$ $e^{2x} = \frac{0}{2}$	$2\cos 2x - \sin 2x = 0$
$e^{2x} = 0$	$2\cos 2x - \sin 2x = 0$ $- \sin 2x = -2\cos 2x$
$x = \frac{1}{2} \pi$ $= 0.6931 \text{ (Rejected)}$ $= 0.69$	$\frac{\sin 2x}{\cos 2x} = 2$ $\tan 2x = 2$
$\frac{2}{1} = \tan^{-1}(2)$	<p>M1</p>
$2x = 116.56, 296.56$	<p>AD</p>
$x = 58.28, 148.28$ $x = 58.3, 148.3$	<p>(3)</p>

Total mark awarded = 3 out of 4

## Example candidate response – 2

Question  
No.

$$2) \quad y = \frac{\sin 2x}{e^{2x}}$$

$$u = \sin 2x$$

$$\frac{du}{dx} = \cos 2x$$

$$v = e^{2x}$$

$$\frac{dv}{dx} = 2e^{2x}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$M1 \quad = \frac{e^{2x} \cdot \cos 2x - \sin 2x \cdot 2e^{2x}}{(e^{2x})^2}$$

A0

$$= \frac{e^{2x} \cos 2x - 2e^{2x} \sin 2x}{(e^{2x})^2}$$

$$= \frac{e^{2x} (\cos 2x - 2 \sin 2x)}{(e^{2x})^2} = \frac{\cos 2x - 2 \sin 2x}{e^{2x}}$$

At stationary point;  $\frac{dy}{dx} = 0$ .

$$\cancel{e^{2x}} (\cos 2x - 2 \sin 2x) = 0$$

M0

$$\{e^{2x}\}$$

$$\frac{2 \sin x}{e^{2x}} = 0 \quad \wedge$$

①

Total mark awarded = 1 out of 4

### Examiner comment

Most candidates chose to differentiate the given expression as a quotient, but differentiation of a product would have been equally acceptable.

Both candidates chose to differentiate the given expression as a quotient with candidate 1 obtaining a completely correct expression initially. A transfer of work from one page to the next resulted in an incorrect factorisation of the numerator and the loss of a factor of  $e^{2x}$  in the denominator. An incorrect differentiation of the trigonometric term meant that candidate 2 was not awarded the accuracy mark.

Both candidates realised that the derivative had to be equated to zero, with candidate 1 using a correct approach to solving the equation formed by equating the trigonometric factor to zero. Candidate 2, however, did not make a valid attempt to solve the trigonometric factor equated to zero.

### Question 3

3 The polynomial  $x^4 - 4x^3 + 3x^2 + 4x - 4$  is denoted by  $p(x)$ .

(i) Find the quotient when  $p(x)$  is divided by  $x^2 - 3x + 2$ . [3]

(ii) Hence solve the equation  $p(x) = 0$ . [3]

#### Mark scheme

- 3 (i) Attempt division by  $x^2 - 3x + 2$  or equivalent, and reach a partial quotient of  $x^2 + kx$  M1  
 Obtain partial quotient  $x^2 - x$  A1  
 Obtain  $x^2 - x - 2$  with no errors seen A1 [3]
- (ii) Correct solution method for either quadratic e.g. factorisation M1  
 One correct solution from solving quadratic or inspection B1  
 All solutions  $x = 2, x = 1$  and  $x = -1$  given and no others A1 [3]

## Example candidate response – 1

HO

①

3)  $p(x) : x^4 - 4x^3 + 3x^2 + 4x - 4$ .

$x^2 - x - 2$  ← Quotient.

$x^2 - 3x + 2$  |  $x^4 - 4x^3 + 3x^2 + 4x - 4$ .

$x^4 - 3x^3 + 2x^2$  -

$-x^3 + x^2 + 4x - 4$ .

$-x^3 + 3x^2 - 2x$  -

$-2x^2 + 6x - 4$  ✓

$-2x^2 + 6x - 4$  -

0

3

ii)  $p(x) = 0$ .

MI  $(x^2 - 3x + 2)(x^2 - x - 2)$

AO  $(x-2)(x-1)(x-2)(x+1)$  ✓

④

Item marks awarded: (i) 3/3; (ii) 1/3

Total mark awarded = 4 out of 6

Example candidate response – 2

Question No.		Nothing is to be written in this margin
(2)	(i) $p(x) = x^4 - 4x^3 + 3x^2 + 4x - 4$	
	$x^2 + x + 4$	M1
	$x^2 - 3x + 2$   $x^4 - 4x^3 + 3x^2 + 4x - 4$	A0
	$x^4 - 3x^3 + 2x^2$ -	A0
	$x^3 + x^2 + 4x - 4$	A0
	$x^3 - 3x^2 + 2x$ -	
	$4x^2 + 2x - 4$	
	$4x^2 - 12x + 8$	
	$-10x - 12$	
(ii)	$p(x) = 0$	
	$x^2 + x + 4$	
	$a=1, b=1, c=4$	
	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$\frac{-1 \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$	
	$\frac{-1 \pm \sqrt{-15}}{2}$	
	$\frac{-1 + 3.87}{2}$ or $\frac{-1 - 3.87}{2}$	
	$1.43$ or $-2.43$	
(iii)	$p(x) = 0$	
	$x^2 - 3x + 2 = 0$ $a=1, b=-3, c=2$	
	$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(2)}}{2(1)}$	
	$\frac{3 \pm \sqrt{1}}{2}$	B1
	$\frac{3+1}{2}$ or $\frac{3-1}{2}$	M1
	$2$ or $1$	A0
	$\therefore (x+2)(x+1)$ 1SW	
		3

Item marks awarded: (i) 1/3; (ii) 2/3

Total mark awarded = 3 out of 6

Examiner comment

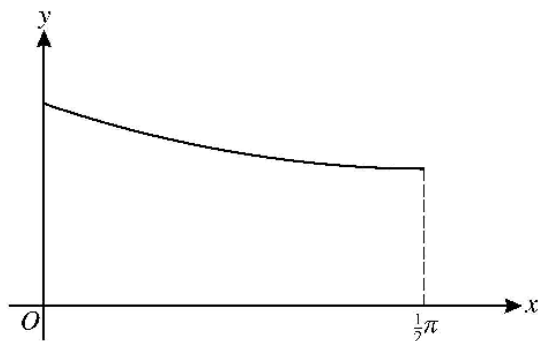
- (i) Algebraic long division was the most popular way of dealing with this part of the question. Both candidates attempted this, with candidate 1 obtaining a perfectly correct division and hence a correct quotient. A slip in subtraction for candidate 2 meant that although a correct method had been employed, no accuracy marks could be obtained.
- (ii) Candidate 1 realised that  $p(x)$  was a product of the given divisor and the quotient found in part (i) and correctly completed the factorisation of both quadratic terms. However, the candidate did not complete the question by solving  $p(x) = 0$ , as required.

Although candidate 2 did not obtain a correct quotient for part (i), they realised that  $x^2 - 3x + 2$  was possibly a factor of  $p(x)$  and were able to go on to obtain two correct solutions using a correct method. A correct quotient from part (i) was needed in order to obtain the other solutions, one of which was a repeated solution. Although the candidate did not obtain a remainder of zero in the first part of the question, they were not penalised again for this in part (ii) when they assumed that  $x^2 - 3x + 2$  was a factor of  $p(x)$ .



## Question 4

4



The diagram shows the part of the curve  $y = \sqrt{2 - \sin x}$  for  $0 \leq x \leq \frac{1}{2}\pi$ .

- (i) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \sqrt{2 - \sin x} \, dx,$$

giving your answer correct to 2 decimal places.

[3]

- (ii) The line  $y = x$  intersects the curve  $y = \sqrt{2 - \sin x}$  at the point  $P$ . Use the iterative formula

$$x_{n+1} = \sqrt{2 - \sin x_n}$$

to determine the  $x$ -coordinate of  $P$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

## Mark scheme

- 4 (i) State or imply correct ordinates 1.4142..., 1.1370..., 1 B1  
 Use correct formula, or equivalent, correctly with  $h = \frac{\pi}{4}$  and three ordinates M1  
 Obtain answer 1.84 with no errors seen A1 [3]
- (ii) Use the iterative formula correctly at least once M1  
 Obtain final answer 1.06 A1  
 Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (1.055, 1.065) B1 [3]

## Example candidate response – 1

(6)

$$y = \sqrt{(2 - \sin x)} \text{ for } 0 \leq x \leq \frac{1}{2}\pi.$$

$$(i) \int_0^{\frac{1}{2}\pi} \sqrt{(2 - \sin x)} dx.$$

$$\text{Width} = \frac{1}{2}\pi - 0 = 1.57.$$

$$x \quad 0 \quad \frac{1}{4}\pi \quad \frac{1}{2}\pi$$

B1

$$y \quad 1.41 \quad 1.14 \quad 1$$

M0

$$= \left( \frac{1}{2} (1.41 + 1) + 1.14 \right).$$

A0

$$= 2.35.$$

## Example candidate response – 1, continued

Question No.		Nothing is to be written in this margin
ii)	$y = x$ $y = \sqrt{2 - \sin x}$ at $p$ . $x_{n+1} = \sqrt{2 - \sin x_n}$	
	$x_1 = 0.7854$ $x = \frac{1}{2}\pi + 0$ $\phantom{x_1} = 0.79$	
	$x_2 = \sqrt{2 - \sin(0.79)}$	
	$x_3 = 1.1356$ <del>1.14</del> $\phantom{x_3} = \sqrt{2 - \sin(1.1356)}$	
	$x_4 = 1.0456$ 1.05	
	$x_5 = \sqrt{2 - \sin(1.0456)}$ $\phantom{x_5} = 1.0653$ 1.07	
	$x_6 = \sqrt{2 - \sin(1.0653)}$ $\phantom{x_6} = 1.0607$ 1.06	
	$x_7 = \sqrt{2 - \sin(1.0607)}$ ✓ 1.06	3
	$\phantom{x_7} = 1.0617$	(4)
	$x$ -coordinate of $p = 1.06$	

Item marks awarded: (i) 1/3; (ii) 3/3

Total mark awarded = 4 out of 6

Example candidate response – 2

Question No.		Nothing is to be written in this margin								
4)	$\text{width} = \frac{\text{upper limit} - \text{lower limit}}{\text{no. of trapezia}}$ $= \frac{\frac{1}{2}\pi - 0}{2}$ $= \frac{\pi}{4}$									
	<table border="0"> <tr> <td><math>x</math></td> <td>0</td> <td><math>\frac{\pi}{4}</math></td> <td><math>\frac{\pi}{2}</math></td> </tr> <tr> <td><math>y</math></td> <td><math>\sqrt{2}</math></td> <td>1.41</td> <td>1.41</td> </tr> </table>	$x$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$y$	$\sqrt{2}$	1.41	1.41	B0
$x$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$							
$y$	$\sqrt{2}$	1.41	1.41							
	<p>Trapezium Rule</p> $= \frac{\pi}{4} \left( \frac{1}{2} (\sqrt{2} + 1.41) + 1.41 \right)$ $= 2.21$	M1 A0								
ii)	$x = \sqrt{2 - \sin^2 x}$ $x_1 = 0.79$ $x_2 = \sqrt{2 - \sin^2 0.79}$ $= 1.4093 \quad (1.41)$	M1								
	$x_3 = \sqrt{2 - \sin^2 1.4093}$ $= 1.4054 \quad (1.41)$	A0								
	$\therefore \text{root} = 1.41$	A0 (2)								

Item marks awarded: (i) 1/3; (ii) 1/3

Total mark awarded = 2 out of 6

### Examiner comment

- (i) For questions of this type, it is important that candidates have their calculator in the correct mode; in this case radian mode. The range of values for  $x$  given in the question is in terms of  $\pi$ , so the use of radians is implied.

Candidate 1 was able to obtain the correct ordinates, but forgot to use the interval width when using the trapezium formula.

Candidate 2 was unable to obtain the correct ordinates as they had their calculator in degree mode. However, they were able to apply the correct formula for the trapezium rule to their three ordinates and thus obtain a method mark.

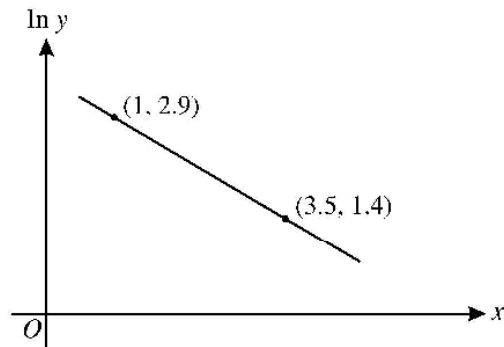
- (ii) Candidates should be encouraged to make full use of the ANS key on their calculator when dealing with iterations, saving time and lessening the chance of miscalculation. Again, candidates should ensure that their calculator is in the correct mode.

A completely correct solution for this part of the question was submitted by candidate 1. Calculations using the ANS function on the calculator were written down correct to 4 decimal places, this accuracy enabling the candidate to make an appropriate deduction of the solution correct to two decimal places after the correct number of iterations.

Due to having their calculator in the incorrect mode, candidate 2 was unable to obtain the accuracy marks available for this part, but was able to be given credit for making use of the iterative formula correctly.

## Question 5

5



The variables  $x$  and  $y$  satisfy the equation  $y = A(b^{-x})$ , where  $A$  and  $b$  are constants. The graph of  $\ln y$  against  $x$  is a straight line passing through the points  $(1, 2.9)$  and  $(3.5, 1.4)$ , as shown in the diagram. Find the values of  $A$  and  $b$ , correct to 2 decimal places. [6]

## Mark scheme

- |   |  |        |
|---|--|--------|
| 5 | State or imply $\ln y = \ln A - x \ln b$                 | B1     |
|   | Form a numerical expression for the gradient of the line | M1     |
|   | Obtain $b = 1.82$  | A1     |
|   | Use gradient and one point correctly to find $\ln A$     | M1     |
|   | Obtain $\ln A = 3.5$                                     | A1     |
|   | Obtain $A = 33.12$                                       | A1 [6] |

## Example candidate response – 1

Question No.		Nothing to be written in this margin
5)	$y = A(b^{-x})$ <p>Take ln on both sides.</p> $\ln y = \ln A + \ln b^{-x}$ $\ln y = \ln A - x \ln b$ $y = mx + c$ $\ln y = -x \ln b + \ln A$	BI
	$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.4 - 2.9}{3.5 - 1} = \frac{-1.5}{2.5} = -0.6$	M1
	$y = mx + c$ $2.9 = 0.6(1) + c$ $c = 2.9 - 0.6 = 2.3$	M1
	$-\ln b = -0.6$ $\ln b = 0.6$ $b = e^{0.6} = 1.82$	A1
	$\ln A = 2.3$	A0
	$A = e^{2.3} = 9.97$	A0
		(4)

Total mark awarded = 4 out of 6

Example candidate response – 2

5.	$y = A(b^{-x})$		
	$\ln y = \ln A(b^{-x})$		
	$\ln y = \ln A + \ln b^{-x}$	(1, 2.9) and (3.5, 1.4)	B1
	$\ln y = -x \ln b + \ln A$	$m = \frac{y_2 - y_1}{x_2 - x_1}$	M1
	$y = mx + c$	$= \frac{1.4 - 2.9}{3.5 - 1} = -1.5$	A0
	$-\ln b = -0.6$	$\ln A = c$	
	$\ln b = 0.6$	$\ln A = 2.9$	M0
	$b = e^{0.6}$	$A = -2.72$	A0
	$= 1.82$		A0
			(2)

Total mark awarded = 2 out of 6

Examiner comment

Both candidates were able to obtain the correct logarithmic form of the given equation, with both also finding the correct gradient. The importance of the negative nature of this gradient cannot be stressed enough as the omission of a negative sign can lead to as many as three accuracy marks not being awarded.

Candidate 1 dealt with the relationship between the gradient and  $\ln b$ , obtaining a correct value for  $b$ . Similar work was done by candidate 2, but a spurious negative sign was introduced in the last two lines of working for the calculation of  $b$ .

Candidate 1 employed a correct approach of using the gradient and a point on the curve in order to find the value of  $\ln A$ , credit being given for this. A sign error in this calculation led to the final two accuracy marks not being awarded.

Candidate 2 did not make use of the gradient and a point on the curve and so was unable to gain any credit for their attempt to find  $A$ .

Candidates should be encouraged to make sure that they substitute in correct values of the variables in this type of question. The coordinates on the given line are for  $x$  and  $\ln y$ , not  $x$  and  $y$ ; a common error made in questions of this type.



## Question 6

6 (a) Find  $\int 4e^{-\frac{1}{2}x} dx$ . [2]

(b) Show that  $\int_1^3 \frac{6}{3x-1} dx = \ln 16$ . [5]

## Mark scheme

- 6 (a) Obtain integral  $ke^{-\frac{1}{2}x}$  with any non-zero  $k$  M1  
 Correct integral A1 [2]
- (b) State indefinite integral of the form  $k \ln(3x-1)$ , where  $k=2, 6$  or  $3$  M1  
 State correct integral  $2 \ln(3x-1)$  A1  
 Substitute limits correctly (must be a function involving a logarithm) M1  
 Use law for the logarithm of a power or a quotient M1  
 Obtain given answer correctly A1 [5]

Example candidate response – 1

6	$\int 4e^{-\frac{1}{2}x} dx$ $= \frac{4e^{-\frac{1}{2}x}}{-\frac{1}{2}}$ $= -2e^{-\frac{1}{2}x} + c$	<p>MI</p> <p>AO</p>
b)	$\int_1^3 \frac{6}{3x-1} dx$ $= \frac{2}{3} \int_1^3 \frac{6}{3x-1} dx$ $= \left[ \frac{6}{3} \ln  3x-1  \right]_1^3$ $\left[ \frac{6}{3} \ln  3(3)-1  - \frac{6}{3} \ln  3(1)-1  \right]$ $\frac{6}{3} \ln 8 - \frac{6}{3} \ln 2$ $2 - 2 \ln 8 - \ln 2$	<p>MI</p> <p>AI</p> <p>MI</p> <p>MO</p> <p>AO</p> <p>(4)</p>

Item marks awarded: (a) 1/2; (b) 3/5

Total mark awarded = 4 out of 7

## Example candidate response – 2

Question No.	
6 a)	$\int 4e^{-1/2x} dx$
	$= \frac{4e^{-1/2x}}{1/2}$
2	$= 8e^{-1/2x} + c$
b)	$\int_1^3 \frac{6}{3x-1} dx = \ln 16$
M1	$= \int_1^3 6 \ln  3x-1 $
A0	
M0	$= [6 \ln  3(3)-1  + 6 \ln  3(1)-1 ]^3$
M0	$= 6 \ln 8 + 6 \ln 2$
M0	$= \ln 8 \times 2$
A0	$= \ln 16$
3	

Item marks awarded: (a) 2/2; (b) 1/5

Total mark awarded = 3 out of 7

Examiner comment

- (a) Both candidates were able to obtain a correct variable term, candidate 2 producing a completely correct response. Candidate 1, while obtaining a correct unsimplified answer, made an arithmetic slip in the final calculation.
- (b) Both candidates recognised that  $\ln(3x - 1)$  was involved; candidate 1 obtained the correct response, with candidate 2 making an arithmetic slip with the associated multiple.

A correct substitution of the limits was made by candidate 1, resulting in a correct statement involving logarithms; however, the appropriate laws of logarithms were not used to obtain the given answer.

Candidate 2 dealt with the limits incorrectly, adding the two terms in the square bracket notation, rather than subtracting them, resulting in a method mark not being awarded. The candidate attempted to use the appropriate law of logarithms, but the '6' was omitted. A method mark could have been awarded at this point had the '6' been written in. The answer for this question was given and the omission of the '6' appeared to give a correct result.

Candidates should be encouraged not to manipulate their work incorrectly in order to obtain given answers. If it appears that the result obtained is not that required, a careful check of previous calculations should be made in an attempt to identify any errors. If these cannot be found, it is much better to leave the work as it is rather than contrive to obtain the given answer (as in this case) as, very often, method marks may be gained.

## Question 7

7 The equation of a curve is

$$3x^2 - 4xy + 2y^2 - 6 = 0.$$

(i) Show that  $\frac{dy}{dx} = \frac{3x - 2y}{2x - 2y}$ . [4]

(ii) Find the coordinates of each of the points on the curve where the tangent is parallel to the  $x$ -axis. [5]

### Mark scheme

- 7 (i) State  $4y \frac{dy}{dx}$  as derivative of  $2y^2$ , or equivalent B1
- State  $4y + 4x \frac{dy}{dx}$  as derivative of  $4xy$ , or equivalent B1
- Equate derivative of LHS to zero and solve for  $\frac{dy}{dx}$  M1
- Obtain given answer correctly A1 [4]
- (ii) State or imply that the coordinates satisfy  $3x - 2y = 0$  B1
- Obtain an equation in  $x^2$  (or  $y^2$ ) M1
- Solve and obtain  $x^2 = 4$  (or  $y^2 = 9$ ) A1
- State answer  $(2, 3)$  A1
- State answer  $(-2, -3)$  A1 [5]

Example candidate response – 1

Question No.		Nothing is to be written in this margin
N <sup>o</sup> 7	$3x^2 - 4xy + 2y^2 - 6 = 0$	
i)	$\text{Show that } \frac{dy}{dx} = \frac{3x - 2y}{2x - 2y}$	
	$3x^2 - 4xy + 2y^2 - 6 = 0$ $6x - (4x \frac{dy}{dx} + y \cdot 4) + 4y \frac{dy}{dx} = 0$ $6x - 4x \frac{dy}{dx} + 4y + 4y \frac{dy}{dx} = 0$ $4x \frac{dy}{dx} + 4y \frac{dy}{dx} = 6x + 4y$ $\frac{dy}{dx} (4x + 4y) = 6x + 4y$	
ii)	$\text{Show that } \frac{dy}{dx} = \frac{3x - 2y}{2x - 2y}$	
	$3x^2 - 4xy + 2y^2 - 6 = 0$	
	$6x - (4x \frac{dy}{dx} + y \cdot 4) + 4y \frac{dy}{dx} = 0$	B1 B1
	$6x - 4x \frac{dy}{dx} + 4y + 4y \frac{dy}{dx} = 0$	M1
	$-4x \frac{dy}{dx} + 4y \frac{dy}{dx} = 6x - 4y$	
	$\frac{dy}{dx} (4x + 4y) = 6x - 4y$	
	$\frac{dy}{dx} = \frac{6x - 4y}{4x + 4y}$	
	$\frac{dy}{dx} = \frac{3x - 2y}{2x - 2y} \quad \text{4 (shown)}$	A0

## Example candidate response – 1, continued

Question No.	Nothing is to be written in this margin
N27	
(ii) Find coordinates of each point where tangent parallel to $x$ -axis. When the tangent is parallel to $x$ -axis $\frac{dy}{dx} = 0$ .	
B1	$3x - 2y = 0$ $2x - 2y$ $3x - 2y = 0$ $3x = 2y$ $x = \frac{2y}{3}$
M1	$3\left(\frac{2y}{3}\right)^2 - 4\left(\frac{2y}{3}\right)y + 2\left(\frac{2y}{3}\right)^2 - 6 = 0$ $4\frac{1}{3}y^2 - 8\frac{1}{3}y^2 + 8\frac{1}{3}y^2 - 6 = 0$ $2y - 8\frac{1}{3}y^2 + 8\frac{1}{3}y^2 - 6 = 0$ $-4\frac{1}{3}y^2 - 6 = 0$
A0	$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $y^2 = 6 + 4\frac{1}{3}$ $y^2 = \frac{58}{3}$
A0	$= -2 \pm \sqrt{4}$
A0	$y = \frac{\sqrt{58}}{3}$
(5)	

Item marks awarded: (i) 3/4; (ii) 2/5

Total mark awarded = 5 out of 9

## Example candidate response – 2

Question No.		Nothing is written in this margin
<u>Q7</u>	$3x^2 - 4xy + 2y^2 - 6 = 0$	
	$6x$	
	$4xy$	
	$u = 4x \quad v = y$	
	$\frac{du}{dx} = 4 \quad \frac{dv}{dx} = \frac{dy}{dx}$	
	$u \frac{dv}{dx} + v \frac{du}{dx}$	
	$4x \cdot \frac{dy}{dx} + y \cdot 4$	
	$4x \cdot \frac{dy}{dx} + 4y$	
B1	$3x^2 - 4xy + 2y^2 - 6 = 0$	
B1	$6x - [4x \cdot \frac{dy}{dx} + 4y] + 4y \frac{dy}{dx} = 0$	
B1	$6x - 4x \frac{dy}{dx} + 4y + 4y \frac{dy}{dx} = 0$	
	$-4x \frac{dy}{dx} + 4y + 4y \frac{dy}{dx} = -6x$	
M1	$-4x \frac{dy}{dx} + 4y \frac{dy}{dx} = -6x - 4y$	
	$\frac{dy}{dx} [-4x + 4y] = -6x - 4y$	
AO	$\frac{dy}{dx} = \frac{-6x - 4y}{4x - 4y}$	
	$= \frac{3x - 2y}{2x - 2y}$	



## Example candidate response – 2, continued

Question No.		Nothing is to be written in this margin
	parallel to x-axis $\frac{dy}{dx} = 0$	
	$\frac{3x-2y}{2x-2y}$	
	$\frac{3x-2y}{3x-2y} = 0$	
	$3x - 2y = 0$	
	$3x - 2 = 0$	
	$3x = 2$	
	$x = \frac{2}{3}$	
	$\therefore$ coordinates $(\frac{2}{3}, 0)$	
		BI
		MO
		↓
		4

Item marks awarded: (i) 3/4; (ii) 1/5

Total mark awarded = 4 out of 9

Examiner comment

- (i) Both candidates were able to use implicit differentiation correctly and obtain correct unsimplified results.

Both used a correct approach of simplification in order to attempt to obtain  $\frac{dy}{dx}$ , but both made a similar sign error in the expansion of the brackets that led to the final accuracy mark not being awarded. It should be noted that the answer was a given answer. Candidates should be encouraged not to manipulate their work incorrectly in order to obtain given answers.

- (ii) Both candidates realised that when  $\frac{dy}{dx} = 0, 3x - 2y = 0$  using the given answer. Questions are often written in this fashion so that candidates may continue with a question even if they have been unable to complete the first part of a question correctly.

Candidate 1 realised that a substitution into the original curve equation needed to be made, but calculation errors gave an incorrect solution, so accuracy marks were not awarded. Candidate 2 interpreted what was required incorrectly and so was unable to gain any further credit for this part of the question.

## Question 8

8 (a) Given that  $\tan A = t$  and  $\tan(A + B) = 4$ , find  $\tan B$  in terms of  $t$ . [3]

(b) Solve the equation

$$2 \tan(45^\circ - x) = 3 \tan x,$$

giving all solutions in the interval  $0^\circ \leq x \leq 360^\circ$ . [6]

## Mark scheme

- 8 (a) Use  $\tan(A + B)$  formula to obtain an equation in  $\tan B$  M1  
 State equation  $\frac{t + \tan B}{1 - t \tan B} = 4$ , or equivalent A1  
 Solve to obtain  $\tan B = \frac{4 - t}{1 + 4t}$  A1 [3]
- (b) State equation  $2 \left( \frac{\tan 45 - \tan x}{1 + \tan 45 \tan x} \right) = 3 \tan x$ , or equivalent B1  
 Transform to a quadratic equation M1  
 Obtain  $3 \tan^2 x + 5 \tan x - 2 = 0$  (or equivalent) A1  
 Solve the quadratic and calculate one angle, or establish that  $\tan x = \frac{1}{3}, -2$  M1  
 Obtain one answer, e.g.  $x = 18.4^\circ$  A1  
 Obtain other 3 answers  $116.6^\circ, 198.4^\circ, 296.6^\circ$  and no others in range A1 [6]

## Example candidate response – 1

$$\text{8(a) } \tan A = t \quad \tan(A+B) = 4.$$

$$\tan(A+B) = 4$$

~~$$\tan A + \tan B = 4$$~~

~~$$t + \tan B = 4.$$~~

~~$$\tan B = 4 - t$$~~

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 4$$

$$\text{M1} \quad \frac{t + \tan B}{1 - t \cdot \tan B} = 4.$$

$$\text{A1}$$

$$4(1 - t \cdot \tan B) = t + \tan B$$

$$4 - 4(\tan B \times t) = t + \tan B$$

$$4 - t - 4t \tan B = \tan B$$

$$\text{A0} \quad \tan B = \frac{4 - t}{4t} \leftarrow$$

## Example candidate response – 1, continued

	$0^\circ < x \leq 360^\circ$	
8b)	$2 \tan(45^\circ - x) = 3 \tan x$	
	$2 \left[ \frac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ \tan x} \right] = 3 \tan x$	BI
	$2 \left( \frac{1 - \tan x}{1 + \tan x} \right) = 3 \tan x$	MI
	$\frac{1 - \tan x}{1 + \tan x} = \frac{3 \tan x}{2}$	
	$3 \tan x (1 + \tan x) = 2(1 - \tan x)$	
	$3 \tan x + 3 \tan^2 x = 2 - 2 \tan x$	
	$3 \tan^2 x + 3 \tan x + 2 \tan x - 2 = 0$	
	$3 \tan^2 x + 5 \tan x - 2 = 0$	AI
	let $\tan x$ be $y$	
	$3y^2 - 5y - 2 = 0$	
	$3y^2 - 6y + 1y - 2 = 0$	MI
	$3y(y-2) + 1(y-2) = 0$	
	$(y-2)(3y+1) = 0$	
	either $y = 2$ or $y = -\frac{1}{3}$	

## Example candidate response – 1, continued

	$\tan x = 2$	$\tan x = -\frac{1}{3}$	
A0	key angle = $\tan^{-1} 2$ = 63.43	= $\tan^{-1} \left(-\frac{1}{3}\right)$ = 18.43	
A0	$x = 63.43, 243.43, 161.57, 341.57$		$\frac{\hat{B}}{\hat{C}} \bigg  \frac{\hat{A}}{\hat{C}}$
(b)			

Item marks awarded: (a) 2/3; (b) 4/6

Total mark awarded = 6 out of 9

## Example candidate response – 2

$$\begin{aligned} \text{a)} \quad \tan A = t \quad \tan (A+B) &= 1 \\ \frac{\tan A + \tan B}{1 - \tan A \tan B} &= 1 \\ \text{M1} \quad \frac{t + \tan B}{1 - t \tan B} &\rightarrow 1 \\ \text{A1} \quad t + \tan B &= 1 - t \tan B \\ t + \tan B &= 1 - t \tan B \\ \tan B + t \tan B &= 1 - t \end{aligned}$$

Example candidate response – 2, continued

Question No.		Nothing is to be written in this margin
	$2 \tan B = \frac{4-t}{4t}$	
	$\tan B = \frac{4-t}{4t} \div 2$	
	$= \frac{4-t}{4t} \times \frac{1}{2}$	
	$= \frac{4-t}{8t}$	AO
	$\frac{B}{8}$	
(b)	$2 \tan (45^\circ - x) = 3 \tan x$	
	$\frac{2 \tan 45^\circ - 2 \tan x}{1 + 2 \tan 45^\circ \cdot 2 \tan x} = 3 \tan x$	BO
	$\frac{2 - 2 \tan x}{1 + 4 \tan x} = 3 \tan x$	
	$2 - 2 \tan x = 3 \tan x$	
	$2 = 5 \tan x$	
	$2 - 2 \tan x = 18 \tan x^2$	
	$-18 \tan x^2 - 2 \tan x + 2 = 0$	
	$18 \tan^2 + 2 \tan x - 2 = 0$	MI
	let $y = \tan x$	AO
	$18y^2 - 2y - 2 = 0$	
	$9y^2 - y - 1 = 0$	
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$= \frac{1 \pm \sqrt{1^2 - 4(9)(-1)}}{2(9)}$	MI
	$= \frac{1 \pm \sqrt{37}}{18}$	AO
	$\frac{1 + \sqrt{37}}{18} = 0.393 \quad \frac{1 - \sqrt{37}}{18} = -0.282$	AO
	$\therefore \tan x = 0.393 \quad \therefore \tan x = -0.282$	
	$x = \tan^{-1}(0.393) = 21.5^\circ, 201.5^\circ \quad x = 15.7^\circ, 164.3^\circ, 341.2^\circ$	(4)

Item marks awarded: (a) 2/3; (b) 2/6

Total mark awarded = 4 out of 9



**Examiner comment**

- (a) Both candidates used the correct trigonometric identity together with the substitution of  $\tan A = t$  and obtained a correct statement. However, incorrect algebraic manipulation from both candidates meant that the correct result was not obtained.
- (b) For candidate 1, use of the correct trigonometric identity together with  $\tan 45^\circ = 1$ , resulted in a correct quadratic equation in  $\tan x$ . An error in copying by the candidate involving a sign change meant that, although a correct method of solution was applied and given credit, no accuracy marks could be given.

For candidate 2, an apparent attempt at the use of the appropriate trigonometric identity was made, but with errors appearing in the numerator. As these were purely numeric errors, the candidate was able to produce a quadratic equation in  $\tan x$  which was solved using a correct method. This meant that the candidate was able to gain method marks but no accuracy marks.

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